

## Unit 4: Exponential Functions

Name: \_\_\_\_\_

Period: \_\_\_\_\_

## Exponential Function Form

$$Y = ab^x$$

Exponent of x make this an exponential function

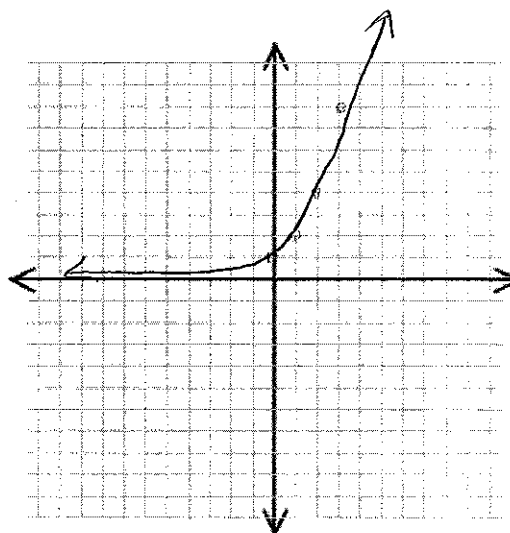
Y-intercept at (0, a)

Growth or Decay

Variable is the exponent, and  $a \neq 1$  while  $b > 0$  excluding 1

### Characteristics

1. Graph  $y = 2^x$
2.  $a = 1$
3.  $b = 2$
4. y-intercept:  $(0, 1)$
5. x-intercept: None
6. Domain:  $(-\infty, \infty)$
7. Range:  $(0, \infty)$
8. Asymptote:  $y = 0$
9. Increasing interval:  $(-\infty, \infty)$
10. Decreasing Interval: none



### Key Attributes:

- Exponential Growth: when  $b > 1$  in the form  $y = ab^x$
- Exponential Decay: when  $0 < b < 1$  in the form  $y = ab^x$

## GUIDED NOTES – Lesson 6-1b

## Translations of Exponential Functions

The equation  $f(x) = (a)b^{x-h} + k$  is the translation function that helps us understand how changing values impacts the resulting graph.

**h** tells us about horizontal movement.

If **h** is positive...

If **h** is negative...

**a** tells us about stretching, reflecting, and compressing.

If **a** < 0...

If **a** > 1...

If  $0 < a < 1$ ...

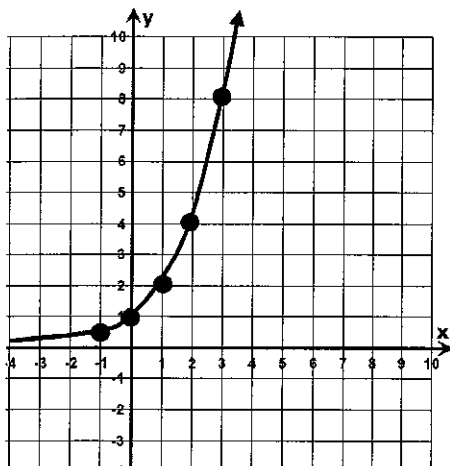
**k** tells us about vertical movement.

If **k** is positive...

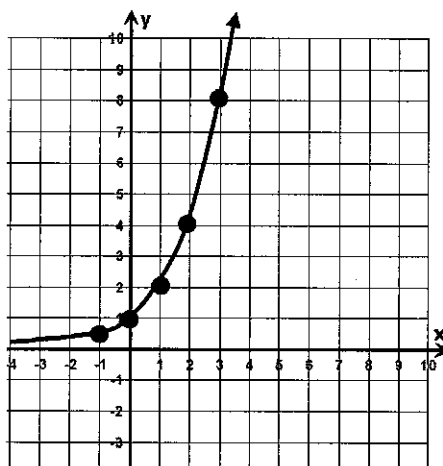
If **k** is negative...

Given the graphed parent function  $f(x) = 2^x$ , perform the following translations.

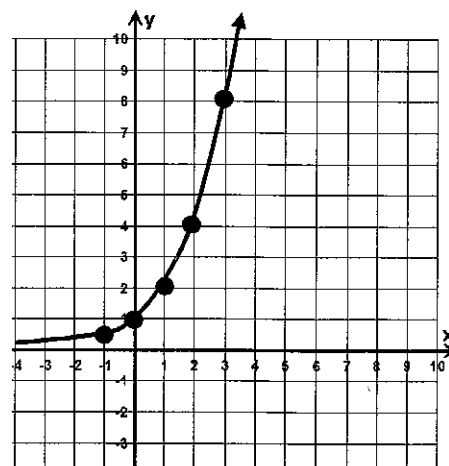
$$f(x) = 2^{x-2}$$



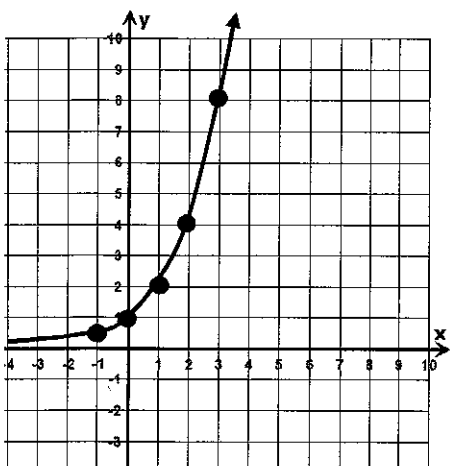
$$f(x) = 2^x - 2$$



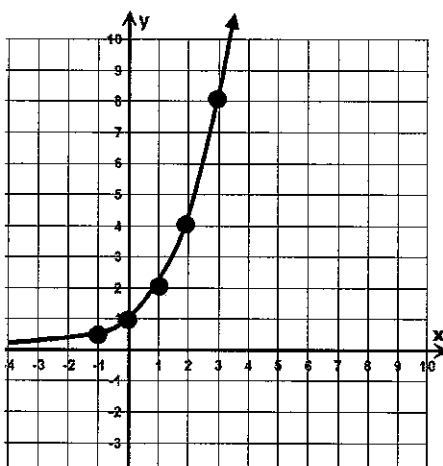
$$f(x) = (-1)2^x$$



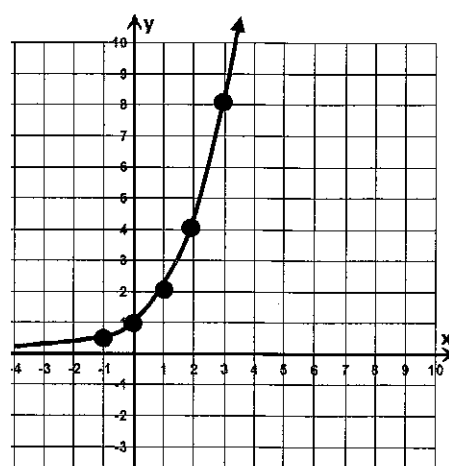
$$f(x) = 2^{x+2}$$



$$f(x) = 2^x + 3$$



$$f(x) = (-2)2^x$$



## Exponential Transformations Worksheet

**1)** Describe the transformations that map the function  $y = 2^x$  onto each of the following functions...

**a)**  $y = 2^x - 2$

**b)**  $y = 2^{x+3}$

**c)**  $y = 4^x$

**d)**  $y = 3(2^{x-1}) + 1$

**2)** Create a sketch of each graph for each equation in question 1. (a table of values may help)

**3)** Write the equation for the function that results from each transformation applied to the base function  $y = 5^x$ .

**a)** translate down 3 units

**b)** shift right 2 units

**c)** translate left  $\frac{1}{2}$  unit

**d)** shift up 1 unit and left 2.5 units

**4)** Write the equation for the function that results from each transformation applied to the base function

$$f(x) = \left(\frac{1}{3}\right)^x$$

**a)** reflect in the x- axis (vertical reflection)

**b)** stretch vertically by a factor of 3

**c)** stretch horizontally by a factor of 2.4

**d)** reflect horizontally, stretch vertically by factor of 4

**5)** Quickly sketch the following exponential functions by transforming the key points and/or asymptote.

**a)**  $y = 3^{x-3} + 2$

**b)**  $y = -\left(\frac{1}{2}\right)^x$

**c)**  $y = \frac{1}{2}(2^x) - 3$

**d)**  $y = \left(\frac{1}{3}\right)^{-2x}$

# **Matching Exponential Graphs and Equations:**

Name: \_\_\_\_\_

**Directions:** Match each equation first with a description of the transformations of the equation (number match) and then with its graph (letter match).

$y = 3^x$

Number Match: \_\_\_\_\_

Letter Match: \_\_\_\_\_

$y = 3^{-x}$

Number Match: \_\_\_\_\_

Letter Match: \_\_\_\_\_

$y = 3^x - 4$

Number Match: \_\_\_\_\_

Letter Match: \_\_\_\_\_

$y = -3^{x-2}$

Number Match: \_\_\_\_\_

Letter Match: \_\_\_\_\_

$y = -3^{-x}$

Number Match: \_\_\_\_\_

Letter Match: \_\_\_\_\_

$y = 3^x + 1$

Number Match: \_\_\_\_\_

Letter Match: \_\_\_\_\_

$y = -3^x$

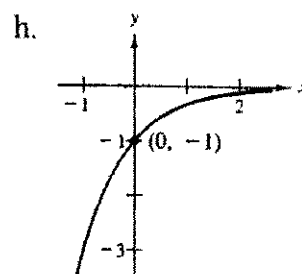
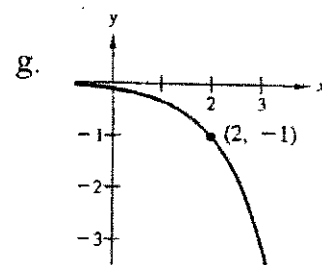
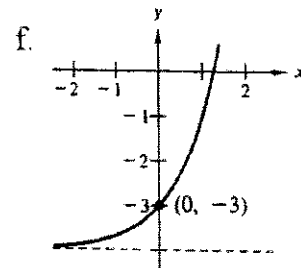
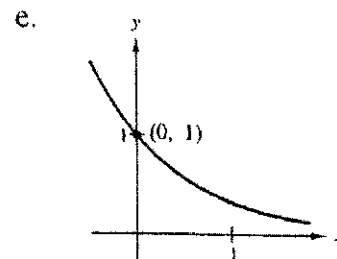
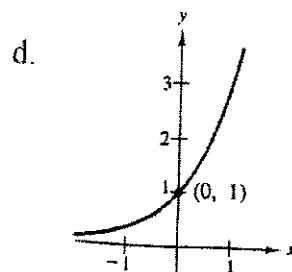
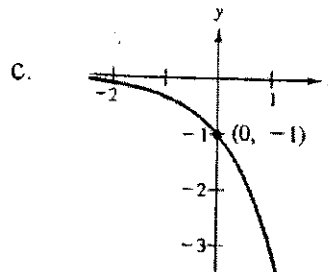
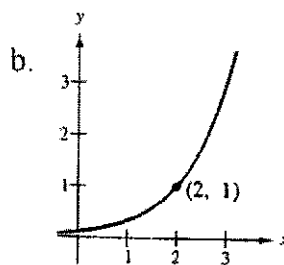
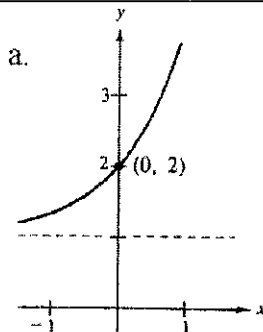
Number Match: \_\_\_\_\_

Letter Match: \_\_\_\_\_

$y = 3^{x-2}$

Number Match: \_\_\_\_\_

Letter Match: \_\_\_\_\_

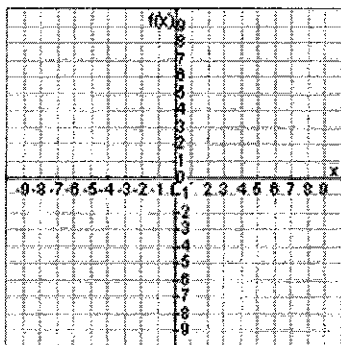


1.  $y = 3^x$  reflected over the x axis and shifted right two
2.  $y = 3^x$  with no transformations
3.  $y = 3^x$  reflected over the y-axis
4.  $y = 3^x$  shifted up 1
5.  $y = 3^x$  shifted right 2
6.  $y = 3^x$  reflected over the x and y axis
7.  $y = 3^x$  shifted down 4
8.  $y = 3^x$  reflected over the x axis

**Find the average rate of change over the given intervals of the following function.**

28.  $f(x) = \left(\frac{1}{2}\right)^x$  on  $[-4, -2]$

$x$					
$f(x)$					

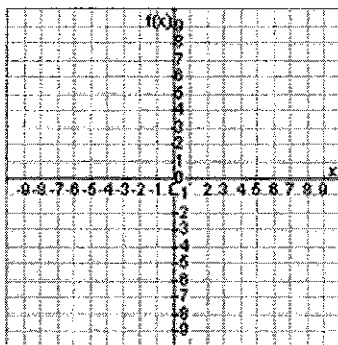


ROC: \_\_\_\_\_

End Behavior:  $\begin{cases} \text{as } x \rightarrow -\infty, f(x) \rightarrow \underline{\hspace{2cm}} \\ \text{as } x \rightarrow \infty, f(x) \rightarrow \underline{\hspace{2cm}} \end{cases}$

29.  $f(x) = \left(\frac{1}{2}\right)^x$  on  $[-2, 2]$

$x$						
$f(x)$						

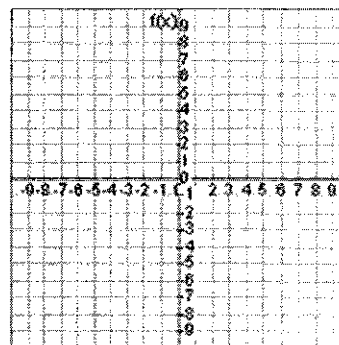


ROC: \_\_\_\_\_

End Behavior:  $\begin{cases} \text{as } x \rightarrow -\infty, f(x) \rightarrow \underline{\hspace{2cm}} \\ \text{as } x \rightarrow \infty, f(x) \rightarrow \underline{\hspace{2cm}} \end{cases}$

30.  $f(x) = \left(\frac{1}{2}\right)^x$  on  $[2,4]$

$x$					
$f(x)$					

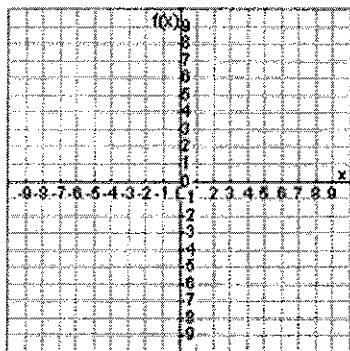


ROC: \_\_\_\_\_

End Behavior:  $\begin{cases} \text{as } x \rightarrow -\infty, f(x) \rightarrow \underline{\hspace{2cm}} \\ \text{as } x \rightarrow \infty, f(x) \rightarrow \underline{\hspace{2cm}} \end{cases}$

31.  $f(x) = 2^x$  on  $[-4, -2]$

$x$					
$f(x)$					

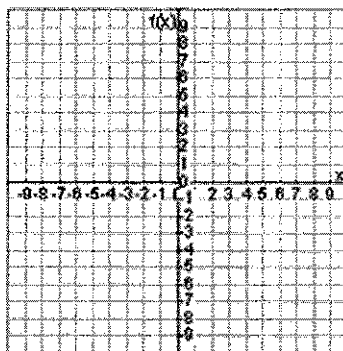


ROC: \_\_\_\_\_

End Behavior:  $\begin{cases} \text{as } x \rightarrow -\infty, f(x) \rightarrow \underline{\hspace{2cm}} \\ \text{as } x \rightarrow \infty, f(x) \rightarrow \underline{\hspace{2cm}} \end{cases}$

32.  $f(x) = 2^x$  on  $[0, 2]$

$x$					
$f(x)$					

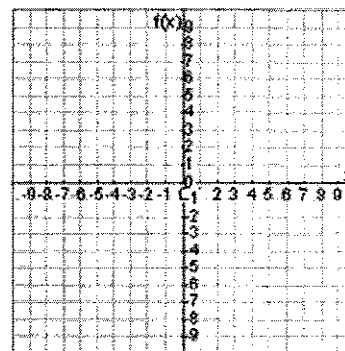


ROC: \_\_\_\_\_

End Behavior:  $\begin{cases} \text{as } x \rightarrow -\infty, f(x) \rightarrow \underline{\hspace{2cm}} \\ \text{as } x \rightarrow \infty, f(x) \rightarrow \underline{\hspace{2cm}} \end{cases}$

33.  $f(x) = 2^x$  on  $[2, 4]$

$x$					
$f(x)$					



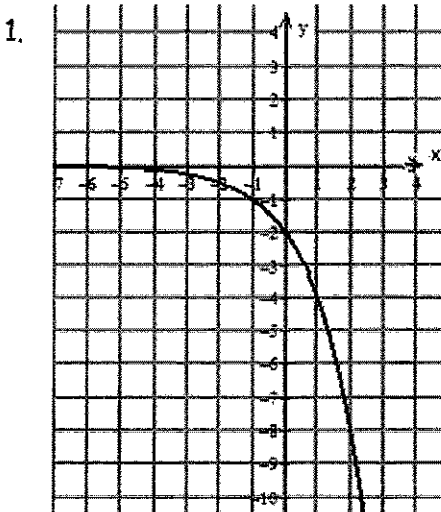
ROC: \_\_\_\_\_

End Behavior:  $\begin{cases} \text{as } x \rightarrow -\infty, f(x) \rightarrow \underline{\hspace{1cm}} \\ \text{as } x \rightarrow \infty, f(x) \rightarrow \underline{\hspace{1cm}} \end{cases}$

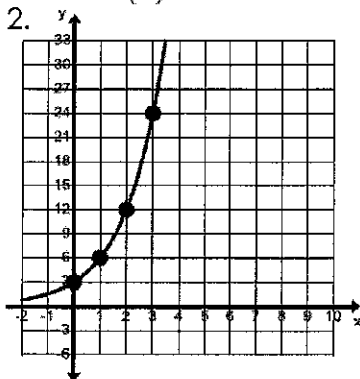
Quiz #2 Review  
Exponential Functions

Name: \_\_\_\_\_

Describe the characteristics of the function below.



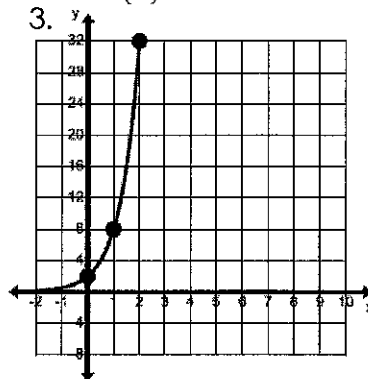
- a. Domain: \_\_\_\_\_  
 b. Range: \_\_\_\_\_  
 c.  $x$ -intercept: \_\_\_\_\_  
 d.  $y$ -intercept: \_\_\_\_\_  
 e. Increasing: \_\_\_\_\_  
 f. Decreasing: \_\_\_\_\_  
 g. Asymptote: \_\_\_\_\_  
 h. End Behavior: \_\_\_\_\_



Rate of Change

[0, 1]

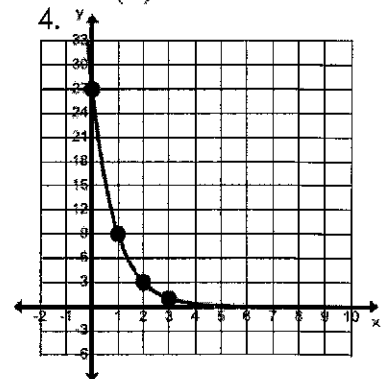
[2, 3]



Rate of Change

[0, 2]

[1, 2]



Rate of Change

[0, 3]

[1, 3]

Find the rate of change over the given intervals for the following function.  $Y = 2^x + 6$

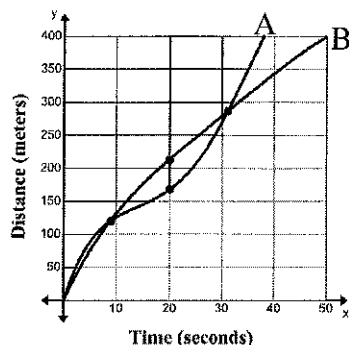
5. [-1, 3] ROC: \_\_\_\_\_

6. [2, 5] ROC: \_\_\_\_\_

## Rate of Change Practice Worksheet

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Below is the graph and table for 2 runners running the 400 meter hurdles race.

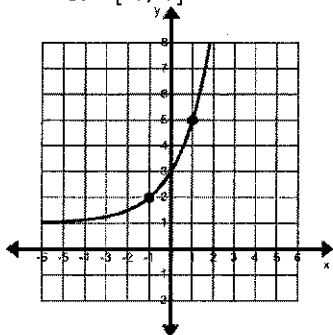


Time	Runner A	Runner B
0	0	0
9	120	120
20	168	213
31	287	287

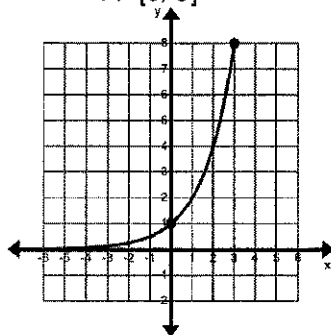
- Which runner has a faster average speed for the first 9 seconds?
- Which runner has a faster average speed from 9 to 20 seconds?
- Which runner has a faster average speed from 20 to 31 seconds?
- Which runner has a faster average speed from 9 to 31 seconds?
- Which runner wins the race? How do you know?

Find the average rate of change for each of the following graphs over the given interval.

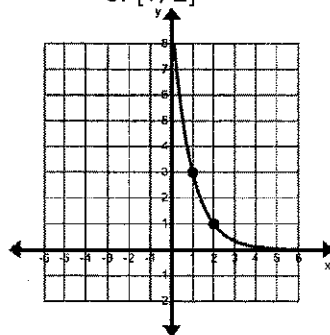
6.  $[-1, 1]$



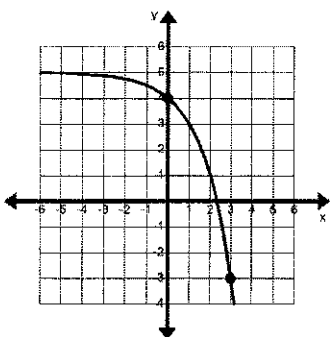
7.  $[0, 3]$



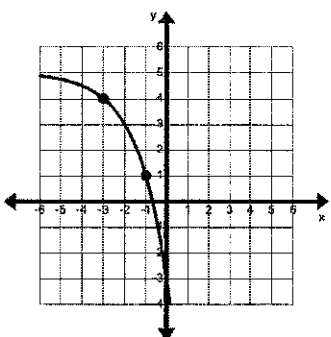
8.  $[1, 2]$



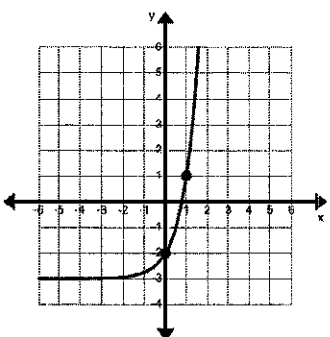
9.  $[0, 3]$



10.  $[-3, -1]$



11.  $[0, 1]$



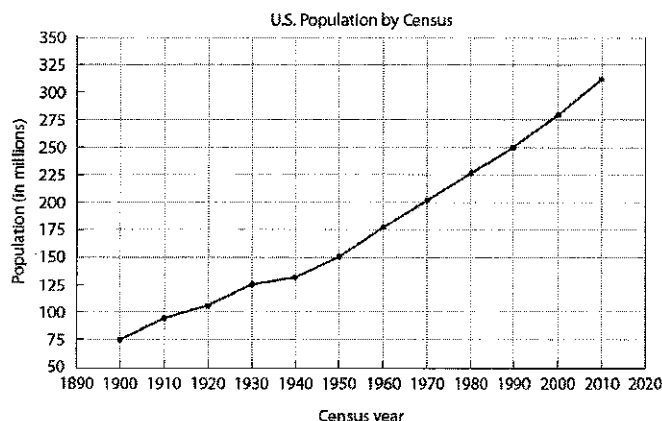


Suppose 25 flour beetles are left undisturbed in a warehouse bin. The beetle population doubles in size every week. The equation  $P(x) = 25 \cdot 2^x$  can be used to determine the number of beetles after  $x$  weeks. Complete the table.

12. Calculate the average growth rate between weeks 1 and 3.
13. Calculate the average growth rate for the first five weeks  $[0, 5]$ .
14. Which average growth rate is higher? Why do you think it is higher?

Week	Population
0	
1	
2	
3	
4	
5	

The graph below shows the United States population from 1900 to 2010, as recorded by the U.S. Census Bureau.



15. What was the rate of change in the population from 1900 to 2000? Is this greater or less than the rate of change in the population from 2000 to 2010?
16. Which 10-year time periods have the highest and the lowest rates of change? How did you find these?

Find the rate of change of Pete's height from 3 to 5 years.

17.

Time (years)	1	2	3	4	5	6
Height(in.)	27	35	37	42	45	49

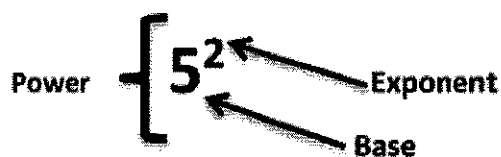
For  $f(x) = x^2 - 2$ , find the rate of change on the interval  $[-2, 4]$ .

18.

# Review of Exponent Operations

Name: \_\_\_\_\_

## Vocabulary:



Exponent Rule	Definition	Example
<b>Product of Powers:</b> $a^m \cdot a^n = a^{m+n}$	If multiplying two numbers with the <b>same base</b> , <b>ADD</b> the exponents.	$y^4 \cdot y^3 \cdot y$  $(7y^5)(6y)$
<b>Quotient of Powers:</b> $\frac{a^m}{a^n} = a^{m-n}$	If dividing two numbers with the <b>same base</b> , <b>SUBTRACT</b> the exponents.	$\frac{6^{13}}{6^2}$  $\frac{10a^7b^9}{15a^5b^9}$
<b>Power of a Power:</b> $(a^m)^n = a^{m \cdot n}$	If raising a power to a power, <b>multiply</b> the exponents.	$(x^2)^8$  $(y^{-3})^{-4}$
<b>Power of a Product:</b> $(ab)^m = a^m b^m$	<b>Distribute</b> power to each factor in parenthesis, and then <b>multiply</b> .	$(4x^3yz)^3$  $(7xy^{-2})^{-2}$
<b>Power of a Quotient:</b> $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	<b>Raise both</b> the numerator and denominator to the power.	$\left(\frac{2a^5}{b^7}\right)^2$
<b>Rules to Remember:</b>	$a^0 = 1$ $a^{-n} = \frac{1}{a^n}$	

**Practice:**

1. $(-3x + 7)^0$	2. $(-3x^2y^7)(5xy^6)$	3. $\left(\frac{a^{-2}b^{-5}}{c^{-11}}\right)^{-6}$
4. $(6x^{-6}y^{-7}z^0)^{-2}$	5. $\left(\frac{3a^{-4}}{b^7}\right)^3$	6. $(y^{-3})^{-4}$

**Applying Exponential Properties to Exponential Equations INTRO:**

1. Write the exponential form of the expression  $2 \times 2 \times 2 \times 2 \times 2 =$  \_\_\_\_\_

2. Evaluate  $2^5 =$  \_\_\_\_\_

3. Therefore  $32 = 2^5$

Rule: In order to solve exponential functions, each power must have the **same base**.

**If  $B^m = B^n$ , then  $m = n$ .**

Consider the following exponential function:  $2^{x+6} = 32$

We can re-write the exponential equation by creating like bases:

$$2^{x+6} = 32$$

$$2^{x+6} = 2^5$$

$$\text{Therefore: } x + 6 = 5$$

$$x = -1$$

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**Practice:**

1.  $9^{2x-5} = 27$

2.  $8^{x-1} = 2^{x+2}$

3.  $36^{2x} = 216^{x-1}$

Solve the Exponential Equation.

1. $5^{3-2x} = 5^{-x}$	2. $3^{2a} = 3^{-a}$
3. $4^{2p} = 4^{-2p-1}$	4. $6^{-2x} = 6^{2-3x}$
5. $8^{x-1} = 2^{x+2}$	6. $144^{2x+1} = 12^{5x-1}$
7. $5^{3x+1} = 25^{x+1}$	8. $4^{-x+1} = 2^{2x}$
9. $64^a = 8^{a+2}$	10. $2^{m+1} = 16^{m+7}$

$$11. 36^{2x} = 216^{x-1}$$

$$12. 3^{a-7} = 27^{2a}$$

$$13. \left(\frac{1}{3}\right)^{x+2} = 3^{x-1}$$

$$14. \left(\frac{1}{2}\right)^x = 2^{x+3}$$

$$15. 8^{x-1} = 2^{x+2}$$

$$16. 8^{5x} = 16^{3x+4}$$

$$17. 81^{2x+1} = 9^{5x-1}$$

$$18. 16^{x+1} = 4^{4x+1}$$

**Warm Up: What makes a function grow exponentially?**

Let's explore: Mr. Smith has an apple orchard. He hires his daughter, Lucy, to pick apples and offers her two payment options.

Option A: \$1.50 per bushel of apples picked

Option B: 1 cent for picking one bushel, 3 cents for picking two bushels, 9 cents for three bushels, and so on, with the amount of money tripling for each additional bushel picked.

- a.) Create a t-chart to model each situation. Let  $x$  represent the number of bushels and  $y$  represent the cost (\$).

Option A

<b>x</b>								
<b>y</b>								

Option B

<b>x</b>								
<b>y</b>								

- b.) If Lucy picks 6 bushels of apples, which option should she choose?
- c.) If Lucy picks 8 bushels of apples, which option should she choose?
- d.) Write a function to model each option.
- e.) Which option would benefit Lucy the most in the long run?
- f.) Describe the differences between Option A and Option B. (How is each situation changing over time? Think ROC)

"b" words indicating base	"a" words	"y" words

**EXPONENTIAL GROWTH/DECAY:** We learned in previous lessons that exponential functions can take on two forms, either growth, when b is \_\_\_\_\_ than 1 or decay, where b is \_\_\_\_\_ 0 and 1.

$$Y = a(1-r)^x$$

$$y' = ab^x$$

$$Y = a(1+r)^x$$

y = final amount

a = initial amount

b = growth/decay

x = time/trials

**Example A:** You are reading a novel where an entire 40 player baseball team become zombies. It is predicted that the number of zombies will triple each day. How many zombies will there be after a week (7 days)?

Seems easy right, but you need to be careful with the variable b, because that example was a nice clean whole number.

**Example B:** You bought a used car for \$18,000. The value of the car will be less each year because of depreciation. The car depreciates (loses value) at the rate of 12% per year. Write an exponential decay model to represent the situation then use that model to estimate the value of the car in 8 years.

**DECAY:**  $A = P(1-r)^t$

**Example C:** A train is going downhill at 140 mph. Suddenly the brake system fails and the train begins picking up speed, going 11% faster every minute. How fast will the train be going in 5 minutes?

**GROWTH:**  $A = P(1+r)^t$

For each problem, write the known information, the unknown, and an equation to model each scenario. Then answer each question.

1. A population of insects doubles every month. If there are 100 insects to start with, how many will there be after 7 months?
2. A stock loses half its value every week. If the stock was worth \$125 starting out, what is it worth after 4 weeks at this rate of decline?
3. A type of bacteria in a Petri dish doubles every hour. If there were 1,073,741,824 bacteria after 24 hours, how many were there to start with?
4. Annual sales for a fast food restaurant are \$650,000 and are increasing at a rate of 4% per year. What will sales be in 5 years?
5. The population of a school is 800 students and is increasing at a rate of 2% per year. What will the population be in 4 years?
6. The value of a company's equipment is \$25,000 and decreases at a rate of 15% each year. What will the equipment be worth after 8 years?



## \* Guided Notes

### Compound Interest & Half-Life

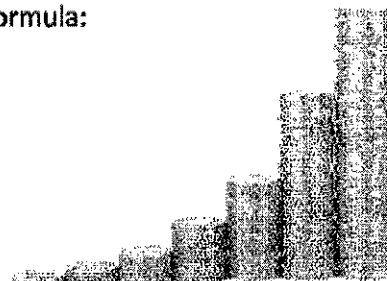
Name: \_\_\_\_\_ Period: \_\_\_\_\_

**OBJECTIVE:** I can apply compound interest formulas and calculate growth and decay in real-world problems.

**COMPOUND INTEREST:** The the balance \_\_\_\_\_ in an account with principal \_\_\_\_\_ and annual interest rate \_\_\_\_\_ (in decimal form) is given by the following formula:

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

For \_\_\_\_\_ compounding per year after \_\_\_\_\_ years.



**Example D:** Find the account balance after 20 years if \$100 is placed in an account that pays 1.2% interest compounded twice a month.

**Example E:** If \$350,000 is invested at a rate of 5% per year, find the amount of the investment at the end of 10 years for the following compounding methods:

a) Quarterly

b) Monthly

**Half Life:** The time required for any specified property to decrease by half. (A measure of decay)

$$A = P(0.5)^t$$

A represents the final amount

P represents the original amount

T represents the number of half-lives in a given time period.

(To find t, divide the time period by the half life)

**Example F:** Fluorine-20 has a half life of 11 seconds. Find the amount of fluorine-20 left from a 40 gram sample after 44 seconds.

**Classwork:**

Find the final amount for each investment.

1. Invested \$1300 earning 5% interest per annum compounded annually for 10 years.
2. Invested \$850 earning 4% interest per annum compounded annually for 6 years.
3. Invested \$720 earning 6.2% interest per annum compounded semiannually for 5 years.
4. Invested \$1100 earning 5.5% interest per annum compounded semiannually for 4 years.
5. Invested \$300 earning 4.5% interest per annum compounded quarterly for 3 years.
6. Invested \$1000 earning 6.5% interest per annum compounded quarterly for 2 years.
7. Invested \$5000 earning 6.3% interest per annum compounded monthly for 10 years.
8. Invested \$2000 earning 5.5% interest per annum compounded daily for 3 years.
9. Bill and Susan's parents want to open a college savings account for their grandchild. They have found an investment that pays 8% annual interest, compounded monthly. How much money will they need to invest in order to have \$60,000 in the account 18 years after their grandchild is born? Round your answer to the nearest dollar.
10. The half-life of plutonium-238 is about 88 years. The amount  $A$  (in grams) of radioactive plutonium-238 that remains in a sample after  $t$  years is given by  $A = 10(.5)^{\frac{t}{88}}$ .  
  
If the original amount of plutonium-238 is 10 grams, how much of the sample will remain
  - a) after 88 years?
  - b) after 176 years?
  - c) after 100 years?

**Modeling with Tic-Tac-Toe:** Choose any three problems to complete following tic-tac-toe order. If you get all three right you win, if not I win! Choose Wisely!

<p>Liza was given \$1000 when she graduated from high school. She decided to invest it in a savings account earning 8% interest compounded annually to put towards her master's degree in the future. How much money will Liza have in ten years? (DOK 2)</p> <p>Equation: _____</p> <p>Solution: _____</p>	<p>River High School is overrun with cockroaches. Resources show that a thriving population of 4400 roaches are crawling through the walls. The exterminator sets up a device to help eliminate the problem which is expected to decrease the population by 12% each week. How many roaches will they have in 6 weeks? (DOK 2)</p> <p>Equation: _____</p> <p>Solution: _____</p>	<p>Sarah's business ended up earning \$10,000 profit its first year open It is expected to triple its profit by 2025. How much will Sarah be making in 2025? (Current year is 2019). (DOK 1)</p> <p>Equation: _____</p> <p>Solution: _____</p>
<p>Elk were re-introduced in north Carolina in 2001. That year, they brought in 25 elk, the following year they brought in 27. Each additional year, the population of elk increased 6%. How many elk can we predict are in North Carolina this year? (DOK 3)</p> <p>Equation: _____</p> <p>Solution: _____</p>	<p>Determine the balance of a \$500 investment at 8% interested compounded quarterly after seven years. (DOK 1)</p> <p>Equation: _____</p> <p>Solution: _____</p>	<p>Brad has accumulated a lot of debt buying video games over the years. He currently has a balance of \$4200 on his credit card but plans to pay half his debt each week. How much will Brad have left in six weeks? (DOK 1)</p> <p>Equation: _____</p> <p>Solution: _____</p>
<p>Randy bought a motorcycle with his first paycheck at his new job. The motorcycle cost \$17,000 and depreciates in value by 7% each year. How much will it be worth in five years when he is ready to trade it in? (DOK 2)</p> <p>Equation: _____</p> <p>Solution: _____</p>	<p>The population of Marietta has increased by 6% since you moved here in 2001. If the current population is 61,048, what was it when you first moved? (Current year is 2019). (DOK 3)</p> <p>Equation: _____</p> <p>Solution: _____</p>	<p>If you invest \$2000 in a savings account with 10% interest compounded monthly, How much will you have in eight years? (DOK 2)</p> <p>Equation: _____</p> <p>Solution: _____</p>

Algebra 1  
Geometric Sequences

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Warm Up

Remember: **Arithmetic Sequences**

**Sequences**

Are the following arithmetic sequences?

If so, what is their common difference?

**Arithmetic**

Recursive:  $a_1 = \underline{\hspace{1cm}}$ ;  $a_n = a_{n-1} + d$

Explicit:  $a_n = a_1 + d(n - 1)$

1. 12, 6, 0, -6, -12, ... \_\_\_\_\_
2. -4, -6, -9, -11, -13 ... \_\_\_\_\_
3. Write the recursive formula given the following sequence: 20, 15, 10, 5, ... \_\_\_\_\_
4. Given the recursive formula, write the explicit formula and find the following terms:  
 $a_1 = 12$ ,  $a_n = a_{n-1} + 7$   
Explicit formula: \_\_\_\_\_  $a_2 = \underline{\hspace{1cm}}$   $a_6 = \underline{\hspace{1cm}}$   $a_9 = \underline{\hspace{1cm}}$
5. Remember: An arithmetic sequence can be modeled by a \_\_\_\_\_ function.

Find the next three terms in the following sequences:

1. 2, 6, 18, 54, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_
2. -3, -9, -27, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

❖ How are these sequences different from arithmetic sequences?

\_\_\_\_\_

❖ To get to the next term in a \_\_\_\_\_ sequence, you must  
\_\_\_\_\_ by a constant.

❖ This constant is called the \_\_\_\_\_.

A \_\_\_\_\_ is a list of terms that are multiplied by a  
\_\_\_\_\_ to get the next term.

Find the common ratio of the following geometric sequences:

3. 4, 16, 64, 256, ... common ratio: \_\_\_\_\_
4. -6, 12, -24, 48, ... common ratio: \_\_\_\_\_
5. 448, 112, 28, 7, ... common ratio: \_\_\_\_\_
6. -288, -144, -72, -36, ... common ratio: \_\_\_\_\_
7. 2000, -200, 20, -2, ... common ratio: \_\_\_\_\_
8. 17, -17, 17, -17, ... common ratio: \_\_\_\_\_

Term # (Position in	1	2	3	4	5
Term Value	2	4	8	16	32

Let's look at a geometric sequence graphed.

Plot the points listed in the chart above.

What is the common ratio for the sequence above?

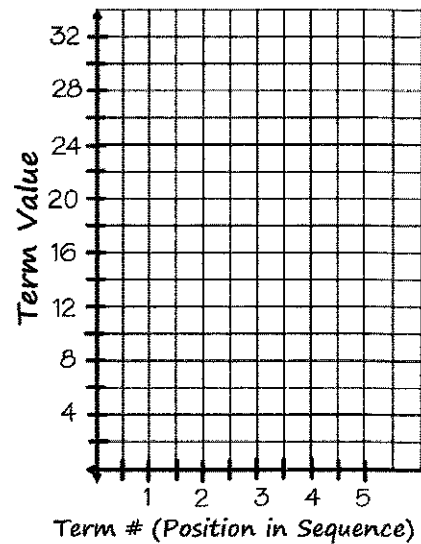
What kind of graph is formed?

Growth or decay?

Should we connect the dots? Why or why not?

Domain:

Range:



So, a geometric sequence **CAN** be modeled by an \_\_\_\_\_ function.

The recursive formula for geometric sequences is similar to the recursive formula for arithmetic sequences.

**Recursive Formula:**  $a_1 = \underline{\hspace{2cm}}, a_n = a_{n-1} \cdot r$

\_\_\_\_\_ is the first term in the sequence \_\_\_\_\_ is the term number

\_\_\_\_\_ is the common ratio \_\_\_\_\_ is the  $n^{\text{th}}$  term in the sequence

**Write the recursive formula for the following sequences.**

1. -4, 8, -16, 32, ... \_\_\_\_\_ 2. 25, 5, 1,  $\frac{1}{5}$ , ... \_\_\_\_\_

**Find the indicated terms given the recursive formula.**

3.  $a_1 = -6, a_n = a_{n-1} \cdot 6$   $a_2 = \underline{\hspace{2cm}}$   $a_3 = \underline{\hspace{2cm}}$   $a_5 = \underline{\hspace{2cm}}$

4.  $a_1 = 32, a_n = a_{n-1} \cdot \frac{1}{2}$   $a_4 = \underline{\hspace{2cm}}$   $a_5 = \underline{\hspace{2cm}}$   $a_6 = \underline{\hspace{2cm}}$

**The Explicit Rule for a Geometric Sequence:**  $a_n = a_1 \cdot (r)^{n-1}$

\_\_\_\_\_ is the first term in the sequence \_\_\_\_\_ is the term number

\_\_\_\_\_ is the common ratio \_\_\_\_\_ is the  $n^{\text{th}}$  term in the sequence

Write the first term, common ratio, and explicit formula for the following geometric sequences.

1. 4, 8, 16, 32,.....

$a_1 = \underline{\hspace{2cm}}$   $r = \underline{\hspace{2cm}}$

$a_n = \underline{\hspace{4cm}}$

2. 68, 34, 17, 8.5,.....

$a_1 = \underline{\hspace{2cm}}$   $r = \underline{\hspace{2cm}}$

$a_n = \underline{\hspace{4cm}}$

Write the explicit formula and find the given term for the following geometric sequence.

3. 3, -9, 27, -81,.....

$a_1 = \underline{\hspace{2cm}}$   $r = \underline{\hspace{2cm}}$

$a_n = \underline{\hspace{4cm}}$

$a_{10} = \underline{\hspace{4cm}}$

4. 24, -12, 6, -3,.....

$a_1 = \underline{\hspace{2cm}}$   $r = \underline{\hspace{2cm}}$

$a_n = \underline{\hspace{4cm}}$

$a_7 = \underline{\hspace{4cm}}$

**Explicit Formula**  
 $a_n = a_1 * (r)^{n-1}$

5. A geometric sequence is defined recursively by  $a_1 = 1$ ;  $a_n = a_{n-1} * (-4)$ .

Find the first 5 terms of the sequence:

Write an explicit formula to represent the sequence:

Find the 10<sup>th</sup> term:

6. A geometric sequence is defined recursively by  $a_1 = 2187$ ;  $a_n = a_{n-1} * (1/3)$ .

Find the first 5 terms of the sequence:

Write an explicit formula to represent the sequence:

Find the 9<sup>th</sup> term:

### Geometric Sequence Word Problem

7. Drenna is participating in a read-a-thon. Her goal is to be reading 128 pages by the time the read-a-thon is over. Drenna reads 2 pages on the first day and then doubles the number of pages she reads every day.

a. Write the first four terms of the Geometric sequence.

b. Write the explicit formula for the Geometric sequence.

c. If the read-a-thon lasts for 8 days, will Drenna have reached her goal of reading 128 pages in a day? How many pages will she be reading on the 8th day?

## Homework

Determine if the sequence is arithmetic or geometric.

1. 10, 30, 90, ...      2. 8, 2, -4, -10, ...

3. -9, -2, 5, ...      4. 625, 125, 25, ...

Geometric Sequences

Recursive:  $a_1 = \underline{\hspace{1cm}}$ ;  $a_n = a_{n-1} \cdot (r)$

Explicit:  $a_n = a_1 \cdot (r)^{n-1}$

Find the first five terms given the first term and the common ratio.

5.  $a_1 = 12$  common ratio: 2      \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

6.  $a_1 = -320$  common ratio:  $-\frac{1}{4}$       \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

For the following geometric sequences, find the common ratio and the missing terms.

7. -1, -7, -49, ...      a. Common ratio: \_\_\_\_\_      b.  $a_5 =$  \_\_\_\_\_      c.  $a_8 =$  \_\_\_\_\_

8. 1024, 512, 256, ...      a. Common ratio: \_\_\_\_\_      b.  $a_5 =$  \_\_\_\_\_      c.  $a_7 =$  \_\_\_\_\_

Write the recursive formula for the following sequences.

9. 99, 33, 11, ...      10. 32, -128, 512, ...

Write the explicit formula for the following sequences.

11. -400, 80, -16, ...      12. 3.5, 21, 126, ...

Given the following formulas, write the first four terms.

13.  $a_1 = -81$ ;  $a_n = a_{n-1} \cdot \left(\frac{1}{3}\right)$

14.  $a_n = 8(3)^{n-1}$

Given the following geometric explicit formulas, write the first term and common ratio, then find the given term.

15.  $a_n = 3072\left(\frac{1}{4}\right)^{n-1}$

16.  $a_n = 3(-5)^{n-1}$

$a_1 =$  \_\_\_\_\_       $r =$  \_\_\_\_\_       $a_7 =$  \_\_\_\_\_       $a_1 =$  \_\_\_\_\_       $r =$  \_\_\_\_\_       $a_8 =$  \_\_\_\_\_

17. The PHS weight training record for the number of push-ups in one day is 507. Jordan wants to beat this record. On the first day, Jordan does one push-up. He then doubles the number of push-ups he does every day.

- Write the first 4 terms of this geometric four sequence.
- Write the explicit formula for the sequence.
- Will Jordan beat the school record after 10 days? Explain.

18. The army is accepting Christmas gifts for kids in need. Their goal is to take 500 toys in one day. The army only receives three toys on the first day. However, on the third day they receive 27 toys.

- How many toys did the army receive on the first days?
- Write the explicit formula for the geometric
- Will they reach their goal by day 6? How many toys will they take in on this day?