Algebra 1
Geometric Sequences

Name: Key
Date: _____

Warm Up
Remember: Arithmetic Sequences

Sequences
Are the following arithmetic sequences? If so, what is their common difference?

Arithmetic
Recursive: $a_1 = \_\_\_$; $a_n = a_{n-1} + d$
Explicit: $a_n = a_1 + d(n-1)$

1. 12, 6, 0, -6, -12, … yes! d = -6 (-6, -6, -6)
2. -4, -6, -9, -11, -13 … NO! (-2, -3, -2, -2)
3. Write the recursive formula given the following sequence: 20, 15, 10, 5, … $a_1 = 20, a_n = a_{n-1} - 5$
   $a_1 = 20$ $d = -5$
4. Given the recursive formula, write the explicit formula and find the following terms:
   $a_1 = 12$, $a_n = a_{n-1} + 7$ $a_n = 12 + 7(n-1)$ $12 + 7n - 7$ $a_6 = 7(6) + 5$ $a_9 = 7(9) + 5$
   Explicit formula: $a_n = 7n + 5$ $a_2 = 19$ $a_6 = 47$ $a_9 = 68$
5. Remember: An arithmetic sequence can be modeled by a linear function.

Find the next three terms in the following sequences:

1. 2, 6, 18, 54, 162, 486, 1458 (×3 ×3 ×3)
2. -3, -9, -27, -81, -243, -729 (×3 ×3)

❖ **How are these sequences different from arithmetic sequences?**
you are multiplying instead of adding or subtracting

❖ **To get to the next term in a** geometric **sequence, you must** multiply **by a constant.**

❖ **This constant is called the** common ratio.

A geometric sequence **is a list of terms that are multiplied by a** common ratio **to get the next term.**

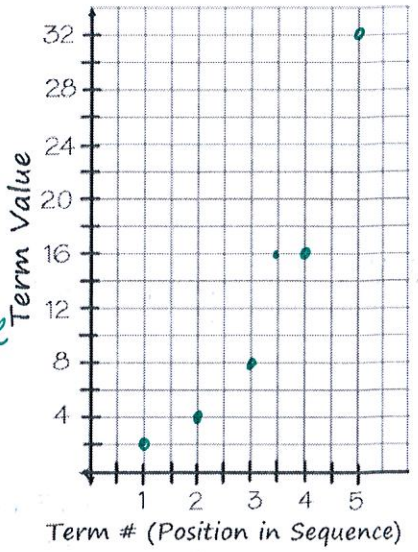
Find the common ratio of the following geometric sequences:

3. 4, 16, 64, 256, … common ratio: r = 4
4. -6, 12, -24, 48, … common ratio: r = -2
5. 448, 112, 28, 7, … common ratio: r = 1/4
6. -288, -144, -72, -36, … common ratio: r = 1/2
7. 2000, -200, 20, -2, … common ratio: r = -1/10
8. 17, -17, 17, -17, … common ratio: r = -1

Term # (Position in	1	2	3	4	5
Term Value	2	4	8	16	32

Let's look at a geometric sequence graphed.

Plot the points listed in the chart above.



What is the common ratio for the sequence above? $r=2$

What kind of graph is formed? exponential

Growth or decay? growth

Should we connect the dots? Why or why not? no — you can't have half a term!

Domain: $\{1,2,3,4,5,\ldots\}$

Range: $\{2,4,8,16,32,\ldots\}$

So, a geometric sequence CAN be modeled by an exponential function.

The recursive formula for geometric sequences is similar to the recursive formula for arithmetic sequences.

Recursive Formula: $a_1 = \_\_\_\_,\ a_n = a_{n-1} \cdot r$

a_1 is the first term in the sequence

n is the term number

r is the common ratio

a_n is the n^{th} term in the sequence

Write the recursive formula for the following sequences.

1. -4, 8, -16, 32, … $a_1=-4;\ a_n=a_{n-1}\cdot(-2)$

2. 25, 5, 1, $\frac{1}{5}$, … $a_1=25;\ a_n=a_{n-1}\cdot\left(\frac{1}{5}\right)$

Find the indicated terms given the recursive formula.

3. $a_1 = -6,\ a_n = a_{n-1} \cdot 6$ $a_2 = -36$ $a_3 = -216$ $a_4 = -1296$ $a_5 = -7776$

4. $a_1 = 32,\ a_n = a_{n-1} \cdot \frac{1}{2}$ $a_2 = 16$ $a_3 = 8$ $a_4 = 4$ $a_5 = 2$ $a_6 = 1$

The Explicit Rule for a Geometric Sequence: $a_n = a_1 * (r)^{n-1}$

a_1 is the first term in the sequence

n is the term number

r is the common ratio

a_n is the n^{th} term in the sequence

Write the first term, common ratio, and explicit formula for the following geometric sequences.

1. 4, 8, 16, 32,.....

$a_1 =$ 4 $r =$ 2

$a_n =$ $4(2)^{n-1}$

2. 68, 34, 17, 8.5,.....

$a_1 =$ 68 $r =$ 1/2

$a_n =$ $68(1/2)^{n-1}$

Write the explicit formula and find the given term for the following geometric sequence.

Explicit Formula
$a_n = a_1 * (r)^{n-1}$

3. 3, -9, 27, -81,.....

$a_1 =$ 3 $r =$ -3

$a_n =$ $3(-3)^{n-1}$

$a_{10} =$ -59049 $3(-3)^{10-1}$

4. 24, -12, 6, -3,.....

$a_1 =$ 24 $r =$ -1/2

$a_n =$ $24(-1/2)^{n-1}$

$a_7 =$ 0.375 $24(-1/2)^{7-1}$

5. A geometric sequence is defined recursively by $a_1 = 1$; $a_n = a_{n-1} * (-4)$.

Find the first 5 terms of the sequence: 1, -4, 16, -64, 256

Write an explicit formula to represent the sequence: $a_n = 1(-4)^{n-1}$

Find the 10th term: $a_{10} = 1(-4)^{10-1} = -262144$

6. A geometric sequence is defined recursively by $a_1 = 2187$; $a_n = a_{n-1} * (1/3)$.

Find the first 5 terms of the sequence: 2187, 729, 243, 81, 27

Write an explicit formula to represent the sequence: $a_n = 2187 \cdot (1/3)^{n-1}$

Find the 9th term: $a_9 = 2187(1/3)^{9-1} = 1/3$

Geometric Sequence Word Problem

7. Drenna is participating in a read-a-thon. Her goal is to be reading 128 pages by the time the read-a-thon is over. Drenna reads 2 pages on the first day and then doubles the number of pages she reads every day.

a. Write the first four terms of the Geometric sequence.

2, 4, 8, 16, ...

b. Write the explicit formula for the Geometric sequence.

$a_n = 2(2)^{n-1}$

c. If the read-a-thon lasts for 8 days, will Drenna have reached her goal of reading 128 pages in a day? How many pages will she be reading on the 8th day?

$a_8 = 2(2)^{8-1} = 256$ She will be reading 256 pages on the 8th day → she does reach her goal!

Homework

Determine if the sequence is arithmetic or geometric.

1. 10, 30, 90, ... geo (x3, x3)
2. 8, 2, -4, -10, ... arithmetic
3. -9, -2, 5, ... arithmetic
4. 625, 125, 25, ... geo (x1/4)

Geometric Sequences

Recursive: $a_1 = \_\_\_$; $a_n = a_{n-1} * (r)$

Explicit: $a_n = a_1 * (r)^{n-1}$

Find the first five terms given the first term and the common ratio.

5. $a_1 = 12$ common ratio: 2 — 12, 24, 48, 96, 192

6. $a_1 = -320$ common ratio: $-\frac{1}{4}$ — -320, 80, -20, 5, -1.25

For the following geometric sequences, find the common ratio and the missing terms.

7. -1, -7, -49, . . . a. Common ratio: 7 b. a_5 = -2401 c. a_8 = -823543

8. 1024, 512, 256, . . . a. Common ratio: 1/2 b. a_5 = 64 c. a_7 = 16

Write the recursive formula for the following sequences.

9. 99, 33, 11, ... $a_1 = 99; a_n = a_{n-1} \cdot (1/3)$
   $a_1 = 99$ $r = 1/3$

10. 32, -128, 512, ... $a_1 = 32, a_n = a_{n-1} \cdot (-4)$
    $a_1 = 32$ $r = -4$

Write the explicit formula for the following sequences.

11. -400, 80, -16, ... $a_n = -400(-1/5)^{n-1}$
    $a_1 = -400$ $r = -1/5$

12. 3.5, 21, 126, ... $a_n = 3.5(6)^{n-1}$
    $a_1 = 3.5$ $r = 6$

Given the following formulas, write the first four terms.

13. $a_1 = -81$; $a_n = a_{n-1} * (1/3)$
    -81, -27, -9, -3

14. $a_n = 8(3)^{n-1}$
    8, 24, 72, 216

Given the following geometric explicit formulas, write the first term and common ratio, then find the given term.

15. $a_n = 3072(1/4)^{n-1}$
    a_1 = 3072 r = 1/4 a_7 = 0.75

16. $a_n = 3(-5)^{n-1}$
    a_1 = 3 r = -5 a_8 = -234375

17. The PHS weight training record for the number of push-ups in one day is 507. Jordan wants to beat this record. On the first day, Jordan does one push-up. He then doubles the number of push-ups he does every day.

    a. Write the first 4 terms of this geometric four sequence.
    1, 2, 4, 8, . . .

    b. Write the explicit formula for the sequence sequence.
    $a_n = 1(2)^{n-1}$

    c. Will Jordan beat the school record after 10 days? Explain.
    $a_{10} = 1(2)^{10-1} = 512$
    Yes!

18. The army is accepting Christmas gifts for kids in need. Their goal is to take 500 toys in one day. The army only receives three toys on the first day. However, on the third day they receive 27 toys.

    a. How many toys did the army receive on the first days?
    3, 9, 27, 81

    b. Write the explicit formula for the geometric
    $a_n = 3(3)^{n-1}$

    c. Will they reach their goal by day 6? How many toys will they take in on this day?
    $a_6 = 3(3)^{6-1} = 729$