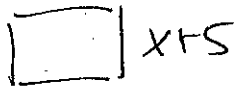
Completing the Square Investigation Name: Answer Key

Georgia Department of Education
Georgia Standards of Excellence Framework

Section 1: Area models for multiplication

1. If the sides of a rectangle have lengths $x + 3$ and $x + 5$, what is an expression for the area of the rectangle? Draw the rectangle, label its sides, and indicate each part of the area.



$A = lw$
$A = (x+3)(x+5)$ FOIL
$x^2+8x+15$

2. For each of the following, draw a rectangle with side lengths corresponding to the factors given. Label the sides and the area of the rectangle:

a. $(x+3)(x+4)$

$x^2+7x+12$

c. $(x-2)(x+5)$

$x^2+3x-10$

b. $(x+1)(x+7)$

x^2+8x+7

d. $(2x+1)(x+3)$

$2x^2+6x+x+3$
$2x^2+7x+3$

Section 2: Factoring by thinking about area and linear quantities

For each of the following, draw a rectangle with the indicated area. Find appropriate factors to label the sides of the rectangle.

1. $x^2 + 3x + 2$

$(x+2)(x+1)$

2. $x^2 + 5x + 4$

$(x+4)(x+1)$

3. $x^2 + 7x + 6$

$(x+6)(x+1)$

4. $x^2 + 5x + 6$

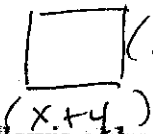
$(x+3)(x+2)$

Section 3: Completing the square

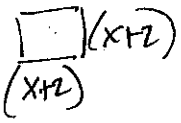
1. What number can you fill in the following blank so that $x^2 + 6x +$ (9) will have two equal factors? What are the factors? Draw the area and label the sides. What shape do you have?

square  $(x+3)$ $(x+3)^2$ $3 \cdot 3 = 3^2$

2. What number can you fill in the following blank so that $x^2 + 8x +$ 16 will have two equal factors? What are the factors? Draw the area and label the sides. What shape do you have?

Square  $(x+4)$ $(x+4)^2$ $4 \cdot 4 = 4^2$

3. What number can you fill in the following blank so that $x^2 + 4x +$ 4 will have two equal factors? What are the factors? Draw the area and label the sides. What shape do you have?

 $(x+2)$ $(x+2)$ Square $(x+2)^2$

4. What would you have to add to $x^2 + 10x$ in order to make a square? What could you add to $x^2 + 20x$ to make a square? What about $x^2 + 50x$? What if you had $x^2 + bx$?

x^2+10x+(25) x^2+20x+(100) x^2+50x+(625) $\left(\frac{b}{2}\right)^2 = c$

Section 4: Solving equations by completing the square

1. Solve $x^2 = 9$ without factoring. How many solutions do you have? What are your solutions?

$\sqrt{x^2} = \sqrt{9}$ $\boxed{x = \pm 3}$

2. Use the same method as in question 5 to solve $\sqrt{(x+1)^2} = \sqrt{9}$. How many solutions do you have? What are your solutions?

$x+1 = \pm 3$
$x = -1 \pm 3$
$\boxed{x = -4, 2}$ *2 solutions

3. In general, we can solve any equation of this form $(x+h)^2 = k$ by taking the square root of both sides and then solving the two equations that we get. Solve each of the following:

a. $\sqrt{(x+3)^2} = \sqrt{16}$
$x+3 = \pm 4$ $x = -3 \pm 4$ $\boxed{x = -7, 1}$

b. $\sqrt{(x+2)^2} = \sqrt{5}$
$x+2 = \pm\sqrt{5}$
$x = -2 \pm \sqrt{5}$ $\boxed{x = -2 \pm \sqrt{5}}$

c. $\sqrt{(x-3)^2} = \sqrt{4}$
$x-3 = \pm 2$
$x = 3 \pm 2$ $\boxed{x = 1, 5}$

d. $\sqrt{(x-4)^2} = \sqrt{3}$
$x-4 = \pm\sqrt{3}$ $\boxed{x = 4 \pm \sqrt{3}}$

4. Now, if we notice that we have the right combination of numbers, we can actually solve other equations by first putting them into this, using what we noticed in questions 1 – 4. Notice that if we have $x^2 + 6x + 9 = 25$, the left side is a square, that is, $x^2 + 6x + 9 = (x + 3)^2$. So, we can rewrite $x^2 + 6x + 9 = 25$ as $(x + 3)^2 = 25$, and then solve it just like we did the problems in question 7. (What do you get?)

$\sqrt{(x+3)^2} = \sqrt{25}$

$x + 3 = \pm 5$ $\quad \boxed{x = -8, 2}$

$x = -3 \pm 5$

5. Sometimes, though, the problem is not written quite in the right form. That's okay. We can apply what we already know about solving equations to write it in the right form, and then we can solve it. This is called completing the square. Let's say we have $x^2 + 6x = 7$. The left side of this equation is not a square, but we know what to add to it. If we add 9 to both sides of the equation, we get $x^2 + 6x + 9 = 16$. Now we can solve it just like the ones above. What is the solution?

$\sqrt{(x+3)^2} = \sqrt{16}$

$x + 3 = \pm 4$ $\quad \boxed{x = -7, 1}$

$x = -3 \pm 4$

6. Try these:

a. $x^2 + 10x = -9$

$\left(\frac{10}{2}\right)^2$

$x^2 + 10x + \underline{25} = -9 + \underline{25}$

$\sqrt{(x+5)^2} = \sqrt{16}$ $\quad x + 5 = \pm 4$

$-5 \quad -5$

$x = -5 \pm 4 = \boxed{-9, -1}$

b. $x^2 + 8x = 20$

$x^2 + 8x + \underline{16} = 20 + \underline{16}$

$\sqrt{(x+4)^2} = \sqrt{36}$ $\quad x + 4 = \pm 6$ $\quad \boxed{x = -10, 2}$

$x = -4 \pm 6$

c. $x^2 + 2x = 5$

$x^2 + 2x + \underline{1} = 5 + \underline{1}$

$\sqrt{(x+1)^2} = \sqrt{6}$ $\quad x + 1 = \pm\sqrt{6}$

$\boxed{x = -1 \pm \sqrt{6}}$

* d. $x^2 + 6x - 7 = 0$

$x^2 + 6x + \underline{9} = 7 + \underline{9}$

$\sqrt{(x+3)^2} = \sqrt{16}$ $\quad x + 3 = \pm 4$ $\quad \boxed{x = -7, 1}$

$x = -3 \pm 4$

* e. $2x^2 + 8x = -6$

$2(x^2 + 4x + \underline{4}) = -6 + \underline{8}$

$\frac{2}{2}(x+2)^2 = \frac{2}{2}$

$\sqrt{(x+2)^2} = \sqrt{1}$ $\quad x + 2 = \pm 1$ $\quad \boxed{x = -3, -1}$

$x = -2 \pm 1$